B.Sc Part II Paper-03
(Hons)

Integral Domain

Integral Domain: — A Commutative ring with unity having more than one element in which the product of non-zero elements is non-zero is called an integral domain.

A ring is said to be with no zero divisor if the product of no two non-zero elements of R is zero ie if $ab=0 \Rightarrow a=0$ or $b=0$.

Ex-1. A ring but not integral domain.

The set E of all even integers with zero is a commutative ring w.r.t. usual addition and multiplication. This ring does not possess unity element for multiplication and hence it is not an integral domain.

Ex-2. The ring of integers I is an integral domain, with respect to addition and multiplication.

Solution: — That the set I of Integers is a Commutative ring with unity has been with respect to usual addion and multiplication. It is an obvious fact that the product of two non-zero integers of I is not zero.

Thus I is an Integral domain.

Theorem - 1. Let D be an integral domain. Let $a, b, c \in D$ where $a \neq 0$ if $a \cdot b = a \cdot c$, then $b = c$.

Proof:— From the distributive law, we have

$$a\cdot(b-c) = a\cdot b - a\cdot c = 0$$

Now since D is an integral domain, therefore from the definition, if $a\cdot(b-c)=0$, then either $a=0$ or $b-c=0$ but $a\neq 0$ and therefore $b-c=0$ which gives $b=c$.

Theorem (2):- If $ax=bx \Rightarrow a=b$; $a,b,x\in D$, then ~~and~~ $ab=0 \Rightarrow a=0$ or $b=0$.

Proof:- Let $a,b\in D$ such that $ab=0$. We ~~to show to~~ show that $a=0$ or $b=0$.

We know that the product of the zero element of any ring with any other element is zero.

Therefore we must have $a\cdot b=0$

Since $a\cdot b=0$, we then have $ab=0b$.

If $b\neq 0$, then from the cancellation law (i.e. $ax=bx \Rightarrow a=b$) we must have $a=0$

Hence either $a=0$ or $b=0$

Proved

~~The purpose of this DO~~

Anjani Kumar Singh.